Refraction Seismic Method

- Intercept times and apparent velocities;
- Critical and crossover distances;
- Hidden layers;
- Determination of the refractor velocity and depth;
- The case of dipping refractor
- Inversion methods:
  - Hagedoorn plus-minus method;
  - Generalized Reciprocal Method;
  - Travel-time continuation.

**Reading:**
- Reynolds, Chapter 5
- Shearer, Chapter 4
- Telford *et al.*, Sections 4.7.9, 4.9
Refraction Seismic Method

- Uses **travel times** of **refracted arrivals** to derive:
  1) Depths to velocity contrasts ("refractors");
  2) Shapes of refracting boundaries;
  3) Seismic velocities.
Apparent Velocity
Relation to wavefronts

- **Apparent velocity**, \( V_{\text{app}} \), is the velocity at which the wavefront sweeps across the geophone spread.

- Because the wavefront also propagates upward, \( V_{\text{app}} \geq V_{\text{true}} \):

  \[
  AC = \frac{BC}{\sin \theta} \quad \rightarrow \quad V_{\text{app}} = \frac{V}{\sin \theta}.
  \]

- 2 extreme cases:
  - \( \theta = 0 \): \( V_{\text{app}} = \infty \);
  - \( \theta = 90^\circ \): \( V_{\text{app}} = V_{\text{true}} \).
Two-layer problem
One reflection and one refraction

At pre-critical offsets, record direct wave and reflection

In post-critical domain, record direct wave, refraction, and reflection

\[ t_{\gamma} = \sqrt{\frac{4h_1^2 + x^2}{V_1}} \]

At pre-critical offsets, record direct wave and reflection

In post-critical domain, record direct wave, refraction, and reflection

Direct: \[ t = \frac{x}{V_1} = x p_1 \]

Reflected: \[ t = t_0 + \frac{x}{V_2} = t_0 + x p_2 \]

Pre-critical

Post-critical

Headwave

Direct

Refracted

\[ V_2 > V_1 \]
Travel-time relations

Two horizontal layers

For a head wave ("often called refraction"):

\[ p = \frac{1}{V_2} \quad \sin i_c = p V_1 \quad \cos i_c = \sqrt{1 - (p V_1)^2} \]

\[ t = 2 \frac{h_1}{V_1 \cos i_c} + p(x - 2h_1 \tan i_c) = t_0 + px \]

\[ t_0 = 2 \frac{h_1}{V_1 \cos i_c} (1 - p V_1 \sin i_c) = \frac{2h_1}{V_1} \cos i_c \]

For a reflection (we'll use this later):

\[ p V_1 = \sin i \]

\[ \tan i = \frac{x}{2h_1} \]

\[ t = 2 \frac{\sqrt{h_1^2 + \left(\frac{x}{2}\right)^2}}{V_1} = \frac{\sqrt{4h_1^2 + x^2}}{V_1} \]

Here, \( p \) is variable and controlled by arbitrary angle \( i \).
Critical and cross-over distances

Critical distance:

\[ x_{\text{critical}} = 2h_1 \tan i_c = 2h_1 \frac{V_1/V_2}{\sqrt{1 - (V_1/V_2)^2}} = \frac{2h_1 V_1}{\sqrt{V_2^2 - V_1^2}} \]

Cross-over distance:

\[ t_{\text{direct}}(x_{\text{crossover}}) = t_{\text{headwave}}(x_{\text{crossover}}) \]

\[ \frac{x_{\text{crossover}}}{V_1} = t_0 + \frac{x_{\text{crossover}}}{V_2} \]

\[ x_{\text{crossover}} = \frac{t_0}{(1/V_1 - 1/V_2)} \]

“slownesses”
Multiple-layer case
(Horizontal layering)

- $p$ is the same critical ray parameter;
- $t_0$ is accumulated across the layers:

$$ p = \frac{1}{V_{\text{refractor}}} $$

$$ t = \sum_{k=1}^{n} \frac{2h_k}{V_k} \cos i_k + px $$

$\sin i_k = pV_k$ in any layer

Critical $p$ determined here
Dipping Refractor Case
shooting down-dip

\[ t = \frac{2h_d}{V_1} \cos i_c + \frac{x}{V_1} \left( \cos \alpha - \sin \alpha \tan i_c \right) + \frac{1}{V_2} \frac{x}{V_1} \sin \alpha \cos i_c \]

\[ t = \frac{2h_d}{V_1} \cos i_c + \frac{x}{V_1} \cos i_c \left[ \frac{V_1}{V_2} (\cos \alpha \cos i_c - \sin \alpha \sin i_c) + \sin \alpha \right] \]

\[ t = \frac{2h_d}{V_1} \cos i_c + \frac{x}{V_1} (\cos \alpha \sin i_c + \sin \alpha \cos i_c) \]

\[ t = \frac{2h_d}{V_1} \cos i_c + \frac{x}{V_1} \sin (i_c + \alpha) \]

would change to '-' for up-dip recording
Refraction Interpretation
Reversed travel times

- One needs *reversed* recording (in opposite directions) for resolution of dips.
- The *reciprocal times*, $T_R$, must be the same for reversed shots.
- Dipping refractor is indicated by:
  - Different *apparent velocities* ($=1/p$, TTC slopes) in the two directions;
    - determine $V_2$ and $\alpha$ (refractor velocity and dip).
  - Different *intercept times*.
    - determine $h_d$ and $h_u$ (interface depths).

\[
\begin{align*}
T & \quad R \\
T_R & \\
\sin (i_c + \alpha) & = \frac{p_d}{V_1} \\
\sin (i_c - \alpha) & = \frac{p_u}{V_1} \\
\frac{2z_d \cos i_c}{V_1} & \\
\frac{2z_u \cos i_c}{V_1} & \\
\text{slope} & = p_1 = \frac{1}{V_1}
\end{align*}
\]
Determination of Refractor Velocity and Dip

- **Apparent velocity** is \( V_{\text{app}} = 1/p \), where \( p \) is the ray parameter (i.e., slope of the travel-time curve).
  - Apparent velocities are measured directly from the observed TTCs;
  - \( V_{\text{app}} = V_{\text{refractor}} \) only in the case of a horizontal layering.
  - For a dipping refractor:
    - Down dip: \( V_d = \frac{V_1}{\sin(i_c + \alpha)} \) (**slower than** \( V_1 \));
    - Up-dip: \( V_u = \frac{V_1}{\sin(i_c - \alpha)} \) (**faster**).

From the two reversed apparent velocities, \( i_c \) and \( \alpha \) are determined:

\[
\begin{align*}
  i_c + \alpha &= \sin^{-1} \frac{V_1}{V_d}, \\
  i_c - \alpha &= \sin^{-1} \frac{V_1}{V_u}.
\end{align*}
\]

\[
\begin{align*}
  i_c &= \frac{1}{2} \left( \sin^{-1} \frac{V_1}{V_d} + \sin^{-1} \frac{V_1}{V_u} \right), \\
  \alpha &= \frac{1}{2} \left( \sin^{-1} \frac{V_1}{V_d} - \sin^{-1} \frac{V_1}{V_u} \right).
\end{align*}
\]

From \( i_c \), the refractor velocity is:

\[ V_2 = \frac{V_1}{\sin i_c}. \]
Determination of Refractor Depth

- From the intercept times, \( t_d \) and \( t_u \), refractor depth is determined:

\[
\begin{align*}
  h_d &= \frac{V_1 t_d}{2 \cos i_c}, \\
  h_u &= \frac{V_1 t_u}{2 \cos i_c}.
\end{align*}
\]
Delay time

Consider a nearly horizontal, shallow interface with strong velocity contrast (a typical case for weathering layer).

- In this case, we can separate the times associated with the source and receiver vicinities: $t_{SR} = t_{SX} + t_{XR}$.

Relate the time $t_{SX}$ to a time along the refractor, $t_{BX}$:

$$t_{SX} = t_{SA} - t_{BA} + t_{BX} = t_{S\text{Delay}} + x/V_2$$

$$t_{S\text{Delay}} = \frac{SA}{V_1} - \frac{BA}{V_2} = \frac{h_s}{V_1 \cos i_c} - \frac{h_s \tan i_c}{V_2} = \frac{h_s}{V_1 \cos i_c} \left(1 - \sin^2 i_c\right) = \frac{h_s \cos i_c}{V_1}.$$  

Note that $V_2 = V_1/\sin i_c$

Thus, source and receiver delay times are:

$$t_{S,R\text{Delay}} = \frac{h_{s,r} \cos i_c}{V_1}.$$  and  $$t_{SR} = t_{S\text{Delay}} + t_{R\text{Delay}} + \frac{SR}{V_2}.$$
Assume that we have recorded two headwaves in opposite directions, and have estimated the velocity of overburden, $V_1$.

How can we map the refracting boundary?

Solution:

- **Profile $S_1 \rightarrow S_2$:**
  \[ t_{S_1D} = \frac{x}{V_2} + t_{S_1} + t_D; \]

- **Profile $S_2 \rightarrow S_1$:**
  \[ t_{S_2D} = \frac{(S_1S_2 - x)}{V_2} + t_{S_2} + t_D. \]

- **Form PLUS travel-time:**
  \[ t_{PLUS} = t_{S_1D} + t_{S_2D} = \frac{S_1S_2}{V_2} + t_{S_1} + t_{S_2} + 2t_D = t_{S_1S_2} + 2t_D. \]

Hence:
  \[ t_D = \frac{1}{2}(t_{PLUS} - t_{S_1S_2}). \]

To determine $i_c$ (and depth), still need to find $V_2$. 
To determine $V_2$:

**Form MINUS travel-time:**

\[ t_{\text{MINUS}} = t_{S_1D} - t_{S_2D} = \frac{2x}{V_2} \frac{S_1S_2}{V_2} + t_{s_1} - t_{s_2}. \]

Hence:

\[ \text{slope}[t_{\text{MINUS}}(x)] = \frac{\frac{2x}{V_2}S_1S_2}{V_2}. \]

- The slope is usually estimated by using the **Least Squares method**.

- **Drawback** of this method – averaging over the pre-critical region.
Generalized Reciprocal Method (GRM)

- Introduces offsets ('XY') in travel-time readings in the forward and reverse shots;
  - so that the imaging is targeted on a compact interface region.
- Proceeds as the plus-minus method;
- Determines the 'optimal' XY:
  1) Corresponding to the most linear velocity analysis function;
  2) Corresponding to the most detail of the refractor.

The velocity analysis function:

\[ t_v = \frac{1}{2} \left( t_{S_1D} - t_{S_2D} + t_{S_1S_2} \right) \]

should be linear, slope \( = 1/V_2 \);

The time-depth function:

\[ t_D = \frac{1}{2} \left( t_{S_1D} + t_{S_2D} - t_{S_1S_2} \frac{XY}{V_2} \right) \]

this is related to the desired depth:

\[ h_D = \frac{t_D V_1 V_2}{\sqrt{V_2^2 - V_1^2}} \]
Head-wave “migration” (travel-time continuation) method

“Migration” refers to transforming the space-time picture (travel-time curves here) into a depth image (position of refractor).

Refraction (head-wave) migration:
- Using the observed travel times, draw the head-wave wavefronts in depth;
- Identify the surface on which:

\[ t_{\text{forward}}(x, z) + t_{\text{reversed}}(x, z) = T_R \]

- This surface is the position of the refractor.
Phantoming

- Refraction imaging methods work within the region sampled by head waves, that is, beyond critical distances from the shots;
- In order to extend this coverage to the shot points, *phantoming* can be used:
  - Head wave arrivals are extended using time-shifted picks from other shots;
  - *However*, this can be done only when horizontal structural variations are small.
Hidden-Layer Problem

- Velocity contrasts *may not be visible* in refraction (first-arrival) travel times. Three typical cases:

  - **Low-velocity layers;**

  - **Relatively thin layers on top of a strong velocity contrast;**

  - **Short travel-time branch may be missed with sparse geophone coverage.**